Model (1)

ALGEBRA &

First Algebra : Answer two only of the following questions

1. (a) Solve in R: ${}^{n+5}P_3 : {}^{n+5}C_3 = \lfloor X^2 - 2X$

(b) Using Cramer's rule, find the solution of the following system of equations: $x + 2z = 5$, $y - 3z - 1 = 0$, $y = 7 - x$

2. (a) put in the simplest form:- $\dfrac{(a\omega + b\omega^2)(a^2\omega^2 - ab + b^2\omega)}{a\omega + b\omega^2 + a\omega^2 + b\omega}$

(b) In the expansion of $\left(2x + \dfrac{3}{x^2}\right)^n$, the ninth and the tenth terms are equal, and the ratio between the sixth term and the seventh term as the ratio 8 **:** 15, find the value of n, then prove that there is no term free of x in this expansion

3. (a) Find the modulus and the principal amplitude of

$Z = \dfrac{1 + i\tan\theta}{1 - i\tan\theta}$ Where $\theta \neq \dfrac{\pi}{2} + k\pi$ and $k \in Z$

(b) Prove that: $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Second Solid geometry : Answer two only of the following questions

4. (a) Complete:-

(i) The two lines in the space which are parallel to a third line ……….
(ii) If a line is parallel to each of two intersecting planes, then ………..
(iii) If two planes are perpendicular to the same straight line then ……
(iv) If L_1 , L_2 are two skew lines, then …………

(b) ABCD is a triangular pyramid, X, Y, Z, L belongs to $\overline{AB}$, $\overline{AC}$, $\overline{DC}$, $\overline{BD}$, respectively such that $\dfrac{AX}{XB} = \dfrac{AY}{YC}$ and $\dfrac{CY}{YA} = \dfrac{CZ}{ZD}$

prove that:- XYZL is a parallelogram.

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5. (a) Prove that: " if a line parallel to a plane, then it is parallel to every line of intersection of this plane with the planes containing the given line "

(b) ABCD is a triangular pyramid, X Î $\overline{CD}$, Y Î $\overline{BC}$, such that each of the two planes AXB and AYD are perpendicular to the plane BCD $\overline{BX} \cap \overline{YD} = \{M\}$, AX = 5 cm, XM = 3 cm, YM = 2 cm, find the measure of the angle of inclination of $\overline{AY}$ on the plane BCD with prove

6. ABCD is a square whose diagonals intersects at M, H is a point outside the plane of the square where HM = MB, if HAB is an equilateral triangle, prove that

(i) $\overline{HM}$ ^ $\overline{MB}$ and plane HAC ^ Plane ABCD

(ii) Find $m\left(H - \overleftrightarrow{AB} - C\right)$

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Answers of model (1)

First Algebra

1. (a) $^{n+5}P_3 \div {}^{n+5}C_3 = \underline{\lfloor X^2 - 2X} \Rightarrow {}^{n+5}P_3 \div \dfrac{^{n+5}P_3}{\underline{\lfloor 3}} = \underline{\lfloor X^2 - 2X}$

$\underline{\lfloor 3} = \underline{\lfloor X^2 - 2X} \Rightarrow X^2 - 2X - 3 = 0 \Rightarrow (X+1)(X-3) = 0$

$X = -1 \quad \text{OR} \quad X = 3 \Rightarrow S.S. = \{-1, 3\}$

(b) $x + 2z = 5 \ , \ y - 3z - 1 = 0 \ , \ y = 7 - x$

$$D = \left|\begin{array}{ccc|cc} \boxed{1} & 0 & 2 & 1 & \boxed{0} \\ 0 & 1 & -3 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{array}\right| = (0+0+0) - (0-3+2) = \boxed{1}$$

$$Dx = \left|\begin{array}{ccc|cc} \boxed{5} & 0 & 2 & 5 & \boxed{0} \\ 1 & 1 & -3 & 1 & 1 \\ 7 & 1 & 0 & 7 & 1 \end{array}\right| = (0+0+2) - (0-15+14) = \boxed{3}$$

$$Dy = \left|\begin{array}{ccc|cc} \boxed{1} & 5 & 2 & 1 & \boxed{5} \\ 0 & 1 & -3 & 0 & 1 \\ 1 & 7 & 0 & 1 & 7 \end{array}\right| = (0-15+0) - (0-21+2) = \boxed{4}$$

$$Dz = \left|\begin{array}{ccc|cc} \boxed{1} & 0 & 5 & 1 & \boxed{0} \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 7 & 1 & 1 \end{array}\right| = (7+0+0) - (0+1+5) = \boxed{1}$$

Then $x = \dfrac{Dx}{D} = 3 \quad , \quad y = \dfrac{Dy}{D} = 4 \quad , \quad z = \dfrac{Dz}{D} = 1$

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2. (a) $= \dfrac{(aw + bw^2)(a^2w^2 - abw^3 + b^2w^4)}{a(w + w^2) + b(w + w^2)} = \dfrac{a^3w^3 + b^3w^6}{-a - b}$

$= \dfrac{a^3 + b^3}{-(a+b)} = \dfrac{(a+b)(a^2 - ab + b^2)}{-(a+b)} = -a^2 + ab - b^2$

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(b) (i) $T_9 = T_{10} \Rightarrow \frac{T_{10}}{T_9} = 1 \Rightarrow \frac{n-9+1}{9} \times \frac{3x^{-2}}{2x} = 1$

$\frac{n-8}{9} \times \frac{3}{2x^3} = 1 \Rightarrow (n-8) = 6x^3 \quad \rightarrow (1)$

$\frac{T_7}{T_6} = \frac{15}{8} \Rightarrow \frac{n-6+1}{6} \times \frac{3x^{-2}}{2x} = \frac{15}{8} \Rightarrow n-5 = \frac{15}{2}x^3 \quad \rightarrow (2)$

By dividing (1) ÷ (2):-

$\frac{n-8}{n-5} = 6x^3 \div \frac{15}{2}x^3 \Rightarrow \frac{n-8}{n-5} = 6 \times \frac{2}{15} = \frac{4}{5}$

$5n - 40 = 4n - 20 \Rightarrow \boxed{n = 20}$

(ii) $T_{r+1} = {}^{n}C_r (2x)^{n-r} (3x^{-2})^r = {}^{n}C_r \times 2^{n-r} \times 3^r \times x^{n-3r}$

$\boxed{T_{r+1} = {}^{20}C_r \times 2^{20-r} \times 3^r \times x^{20-3r}}$

Free term:- $X^0 = X^{20-3r} \Rightarrow 20 - 3r = 0 \Rightarrow r = \frac{20}{3}$, then

There is no free term in this expansion

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3. (a) $Z = \frac{1 + i \times \frac{\sin\theta}{\cos\theta}}{1 - i \times \frac{\sin\theta}{\cos\theta}} \times \left(\frac{\cos\theta}{\cos\theta}\right)$

$Z = \frac{\cos\theta + i \sin\theta}{\cos\theta - i \sin\theta} = \frac{\cos\theta + i \sin\theta}{\cos(2\pi - \theta) + i \sin(2\pi - \theta)}$

$Z = \cos(2\theta - 2\pi) + i \sin(2\theta - 2\pi)$

$Z = \cos(2\pi - 2\theta) - i \sin(2\pi - 2\theta) = \cos 2\theta + i \sin 2\theta$, then

$\boxed{r = 1}$ and $\boxed{P.amp.(Z) = 2\theta}$

<u>Another solution:-</u>

$Z = \frac{1 + i \tan\theta}{1 - i \tan\theta} \times \frac{1 + i \tan\theta}{1 + i \tan\theta} = \frac{(1 + i \tan\theta)^2}{1 + \tan^2\theta} = \frac{1 - \tan^2\theta + 2\tan\theta\, i}{1 + \tan^2\theta}$

$Z = \cos^2\theta \left[1 - \frac{\sin^2\theta}{\cos^2\theta} + 2 \times \frac{\sin\theta}{\cos\theta} i\right] = (\cos^2\theta - \sin^2\theta) + (2\sin\theta\cos\theta)\, i$

$Z = \cos 2\theta + i \sin 2\theta \Rightarrow \boxed{r = 1}$ and $\boxed{P.amp.(Z) = 2\theta}$

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(b) $D = \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$ By adding $[C_1 \times -1 + C_2 \times -1]$ to $[C_3]$

$D = \begin{vmatrix} a+b & b+c & -2b \\ b+c & c+a & -2c \\ c+a & a+b & -2a \end{vmatrix}$ By taking $[-2]$ as a common factor from $[C_3]$

$D = -2\begin{vmatrix} a+b & b+c & b \\ b+c & c+a & c \\ c+a & a+b & a \end{vmatrix}$ By adding $[C_3 \times -1]$ to $[C_1]$

$D = -2\begin{vmatrix} a & b+c & b \\ b & c+a & c \\ c & a+b & a \end{vmatrix}$ By adding $[C_3 \times -1]$ to $[C_2]$

$$D = -2\begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Second Solid geometry

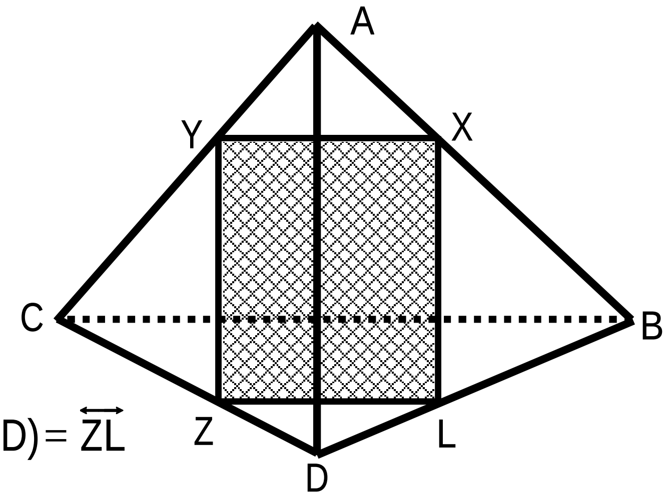
4. (a) (i) are parallel

(ii) It parallel to there line of intersection

(iii) they are parallel

(iv) $L_1 \cap L_2 = \emptyset$ and L_1 , L_2 lie in two different planes

(b) **Proof:**



$\frac{AX}{XB} = \frac{AY}{YC} \Rightarrow \therefore \overline{XY} \parallel \overline{BC}$

$\because \overline{BC} \subset \text{Plane}(BCD)$

$\therefore \overline{XY} \parallel \text{Plane}(BCD)$

$\because \overline{XY} \subset \text{Plane}(XYZL)$

Where plane (XYZL) $\cap$ plane (BCD) $= \overleftrightarrow{ZL}$

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\ $\overline{XY}$ // $\overline{ZL}$ ® (1)

∵ $\frac{CY}{YA} = \frac{CZ}{ZD}$ Þ \ $\overline{YZ}$ // $\overline{AD}$

∵ $\overline{AD}$ Ì Plane(ABD) , \ $\overline{YZ}$ // Plane(ABD)

∵ $\overline{YZ}$ Ì Plane(XYZL) where plane (XYZL) ⋂ plane (ABD) = $\overline{XL}$

\ $\overline{YZ}$ // $\overline{XL}$ ® (2)

from (1) and (2):- \ $\overline{XY}$ // $\overline{ZL}$ and $\overline{YZ}$ // $\overline{XL}$

\ XYZL is a parallelogram

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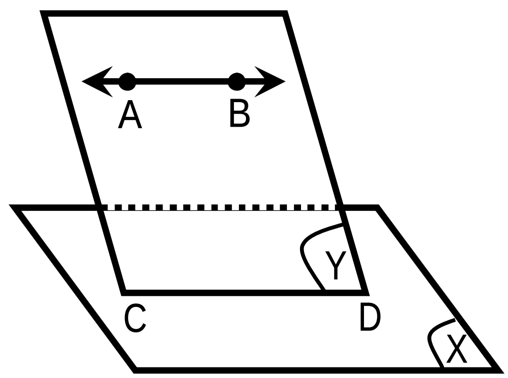
5. (a) Given:



$\overleftrightarrow{AB}$ // X and $\overleftrightarrow{AB}$ Ì Y, where X ⋂ Y = $\overleftrightarrow{CD}$

R.T.P.:

Prove that $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$

Proof:

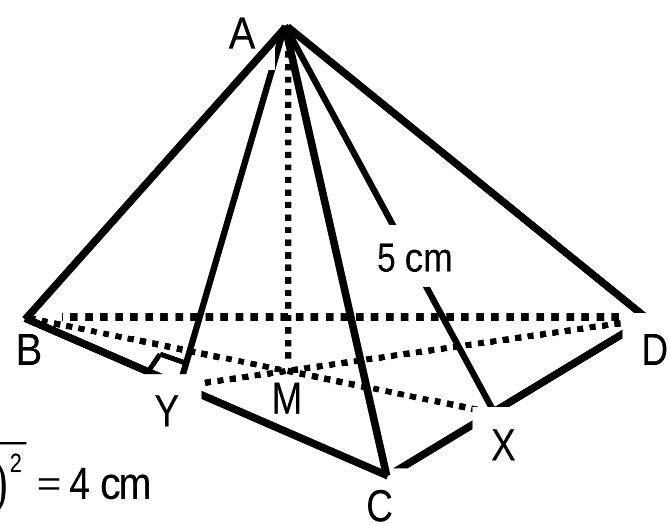
$\overleftrightarrow{AB}$ // X Þ $\overleftrightarrow{AB}$ ⋂ X = f

But $\overleftrightarrow{CD}$ Ì X Þ $\overleftrightarrow{CD}$ ⋂ $\overleftrightarrow{AB}$ = f

Then $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel or skew

But $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ Ì Y Þ $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$

(b) Proof:



∵ Plane(AYD) ^ Plane(BCD)

∵ Plane(ABX) ^ Plane(BCD)

\ $\overline{AM}$ ^ Plane(BCD)

\ $\overline{AM}$ ^ to each of $\overline{DY}$ and $\overline{BX}$

In DAMX

∵ m(AMX) = 90°

\ $AM = \sqrt{(AX)^2 - (MX)^2} = \sqrt{(5)^2 - (3)^2} = 4$ cm

In DAMY

$\tan(AYM) = \frac{AM}{MY} = \frac{4}{2} = 2$ Þ \ m(AYM) = 63°26\

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6. (i) ABCD is a square

$\therefore MA = MB = MC = MD$

$\because MH = MA\ ,\ HB = AB\ ,\ \overline{MB}$ Common

$\therefore \triangle MHB \equiv \triangle MAB$

$\therefore m(HMB) = m(AMB) = 90^\circ$

$\therefore \overline{HM} \perp \overline{MB}$

$AM = MB\ ,\ HA = HB\ ,\ \overline{HM}$ Common

$\therefore \triangle HMA \equiv \triangle HMB$

$\therefore m(HMB) = m(HMA) = 90^\circ$

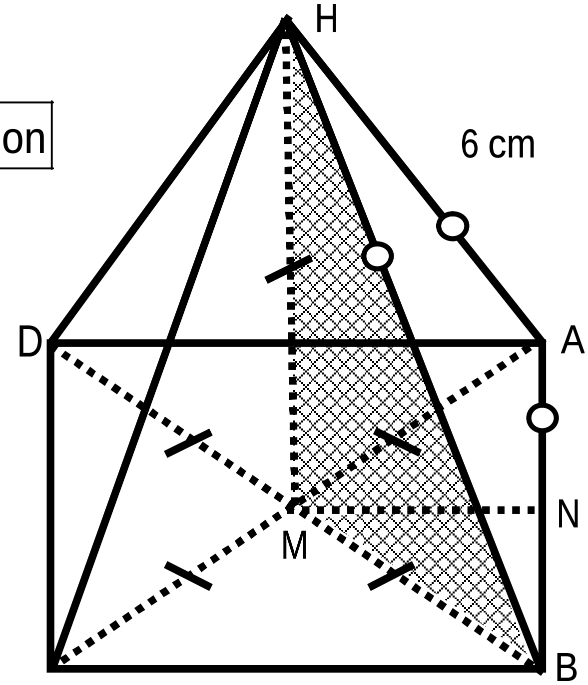
$\therefore \overline{HM} \perp \overline{AM}$

$\therefore \overline{HM} \perp$ each of $\overline{AM}\ ,\ \overline{MB}$

$\therefore \overline{HM} \perp$ Plane ABCD

$\because \overline{HM} \subset$ Plane (AHC)

$\therefore P(HAC) \perp P(ABCD)$



(ii) Take N is a midpoint of $\overline{AB}$

join $\overline{NM}$, $\overline{NH}$

Let the side length of a square equals L

$$MN = \frac{1}{2}AB \quad \Rightarrow \quad MN = \frac{L}{2}$$

$\triangle ABH$ equilateral, $\overline{AN}$ median

$$\therefore \overline{AN} \perp \overline{AB} \quad \Rightarrow \quad HN = L\sin 60 = \frac{L\sqrt{3}}{2}$$

$\angle HNM$ Plane angle of $\angle H - \overleftrightarrow{AB} - C$

$\therefore m\left(\angle H - \overleftrightarrow{AB} - C\right) = m(HNB)$ But

$$\cos(HNM) = \frac{L/2}{L\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$m(HNM) = 54^\circ 44' \quad \Rightarrow \quad m\left(\angle H - \overleftrightarrow{AB} - C\right) = 54^\circ 44'$$

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Model (2)

ALGEBRA &

First Algebra : Answer two only of the following questions

1. (a) Prove that : ${}^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$, then find: $\frac{\left({}^{17}C_{6} + {}^{17}C_{5}\right)}{{}^{18}C_{5}}$

(b) Using Cramer's method, find the solution set of the following system of equations: $2x + y + z = 1, x + 2y + z = 0, x + y + 2z = -1$

2. (a) If "w" is one of the cubic roots of one, prove that:

$$\begin{vmatrix} a+bw & c \\ -1 & w \end{vmatrix}^2 + \begin{vmatrix} w & b \\ -1 & aw+c \end{vmatrix}^2 + \begin{vmatrix} w^4 & a \\ -1 & cw+b \end{vmatrix}^2 = 0$$

(b) In the expansion $\left(\frac{x^3}{2} - 4x^{-1}\right)^{11}$ in descending power of x , find:

(i) The value of coefficient of term contains x^5

(ii) The value of x which makes the sum of the two middle terms equals zero

3. (a) Put the complex number $\left(2 - 2\sqrt{3}\,i\right)$ in the trigonometric form, then find the value of a and b which satisfy $(a + i\,b)^2 = 2 - 2\sqrt{3}\,i$, then prove that : $(a + i\,b)^6 = -64$

(b) If $(x - 2)$ is a factor of the determinant

$$D = \begin{vmatrix} x-1 & x+3 & 2 \\ -3 & x+5 & -6 \\ x+3 & 2 & x+k \end{vmatrix},$$ then find the value of "k".

Second Solid geometry : Answer two only of the following questions

4. (a) Complete:-

(i) If a line not belonging to a plane is parallel to a line in the plane, then……

(ii) the angle between two skew lines is one of the angles between one of them and ………

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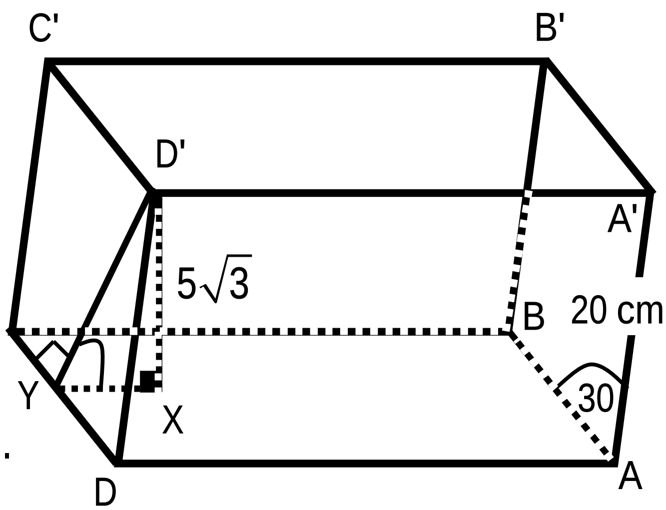
(iii) the sum of the lengths of the diagonals of a rectangular parallelepiped whose dimensions are 15cm, $5\sqrt{3}$ cm and 10cm equals……cm



(b) <u>In the given figure:-</u>
$ABCDA^{\backslash}B^{\backslash}C^{\backslash}D^{\backslash}$ is an inclined parallelepiped where
$\overline{D^{\backslash}X}$ ∧ plane (ABCD),
$\overline{D^{\backslash}Y}$ ∧ $\overline{CD}$, $AA^{\backslash}$ = 20 cm,
$m(A^{\backslash}AB) = 30$, $D^{\backslash}X = 5\sqrt{3}$ cm.
find the measure of dihedral angle between plane ($CC^{\backslash}D^{\backslash}D$) and plane (ABCD)

5. (a) Prove that: " If a line inclined to a plane is perpendicular it, then its projection on the plane is perpendicular to the line in the plane "

(b) $\overline{AB}, \overline{CD}$ are two non coplanar line segments, M is the mid point of $\overline{BD}$, the plane (Mxy) is drawn parallel to each of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ and cuts $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$ at x and y respectively, prove that:

(i) $\overline{My}$ $\overline{AB}$ and $\overline{Mx}$ $\overline{CD}$ (ii) $xy < \frac{1}{2}[AB + CD]$

6. ABC is right angled triangle at B, $\overline{BD}$ is drawn ∧ to plane ABC, and $\overline{DE}$ drawn ∧ $\overline{AC}$ where $\overline{DE} \cap \overline{AC} = \{E\}$, if area of DACD $= 30\ cm^2$, AB = 6 cm, BC = 8 cm

(i) length of $\overline{BD}$

(ii) the tangent of angle between $\overline{DE}$ and plane ABC

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Answers of model (2)

First Algebra

1. (a) $L.H.S.=\dfrac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}\times\dfrac{(r+1)}{(r+1)}+\dfrac{\underline{|n}}{\underline{|r+1}\,\underline{|n-r-1}}\times\dfrac{(n-r)}{(n-r)}$

$$L.H.S.=\frac{(r+1)\underline{|n}}{\underline{|r+1}\,\underline{|n-r}}+\frac{(n-r)\underline{|n}}{\underline{|r+1}\,\underline{|n-r}}=\frac{r\underline{|n}+\underline{|n}+n\underline{|n}-r\underline{|n}}{\underline{|r+1}\,\underline{|n-r}}=\frac{\underline{|n}(n+1)}{\underline{|r+1}\,\underline{|n-r}}$$

$$L.H.S.=\frac{\underline{|n+1}}{\underline{|r+1}\,\underline{|n-r}}={}^{n+1}C_{r+1}=R.H.S.$$

(ii) $\dfrac{{}^{17}C_6+{}^{17}C_5}{{}^{18}C_5}=\dfrac{{}^{18}C_6}{{}^{18}C_5}=\dfrac{18-6+1}{6}=\dfrac{13}{6}$

(b) $2x+y+z=1\ ,\ x+2y+z=0\ ,\ x+y+2z=-1$

$$D=\left|\begin{array}{ccc|cc} \boxed{2} & 1 & 1 & 2 & \boxed{1}\\ 1 & 2 & 1 & 1 & 2\\ 1 & 1 & 2 & 1 & 1\end{array}\right.=(8+1+1)-(2+2+2)=\boxed{4}$$

$$Dx=\left|\begin{array}{ccc|cc} \boxed{1} & 1 & 1 & 1 & \boxed{1}\\ 0 & 2 & 1 & 0 & 2\\ -1 & 1 & 2 & -1 & 1\end{array}\right.=(4-1+0)-(0+1-2)=\boxed{4}$$

$$Dy=\left|\begin{array}{ccc|cc} \boxed{2} & 1 & 1 & 2 & \boxed{1}\\ 1 & 0 & 1 & 1 & 0\\ 1 & -1 & 2 & 1 & -1\end{array}\right.=(0+1-1)-(2-2+0)=\boxed{0}$$

$$Dz=\left|\begin{array}{ccc|cc} \boxed{2} & 1 & 1 & 2 & \boxed{1}\\ 1 & 2 & 0 & 1 & 2\\ 1 & 1 & -1 & 1 & 1\end{array}\right.=(-4+0+1)-(-1+0+2)=\boxed{-4}$$

Then $x=\dfrac{Dx}{D}=1\quad,\quad y=\dfrac{Dy}{D}=0\quad,\quad z=\dfrac{Dz}{D}=-1$

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2. (a) $\text{L. H. S.} = \begin{vmatrix} a+b\omega & c \\ -1 & \omega \end{vmatrix}^2 + \begin{vmatrix} \omega & b \\ -1 & a\omega+c \end{vmatrix}^2 + \begin{vmatrix} \omega^4 & a \\ -1 & c\omega+b \end{vmatrix}^2$

$$\text{L. H. S.} = \left(a\omega + b\omega^2 + c\right)^2 + \left(a\omega^2 + c\omega + b\right)^2 + \left(c\omega^2 + b\omega + a\right)^2$$

$$\text{L. H. S.} = \omega^2\left(a + b\omega + c\omega^2\right)^2 + \omega^4\left(a + b\omega + c\omega^2\right)^2 + \left(a + b\omega + c\omega^2\right)^2$$

$$\text{L. H. S.} = \left(a + b\omega + c\omega^2\right)^2 \left[\omega^2 + \omega + 1\right] = \left(a + b\omega + c\omega^2\right)^2 \times 0 = 0 = \text{R. H. S.}$$

(b) The expansion $\left(\frac{x^3}{2} - \frac{4}{x}\right)^{11}$

$F = 2^{-1} \times x^3$
$S = -4x^{-1}$
$n = 11$

$$T_{r+1} = \left({}^{n}C_r\right) \times \left(F^{n-r}\right) \times \left(S^r\right) = {}^{11}C_r \left(2^{-1}x^3\right)^{11-r} \left(-4x^{-1}\right)^r$$

$$T_{r+1} = {}^{11}C_r \left(2\right)^{-11+r} x^{33-3r} \times \left(-1\right)^r \times 2^{2r} \times x^{-r}$$

$$\boxed{T_{r+1} = \left(-1\right)^r\, {}^{11}C_r \left(2\right)^{3r-11} x^{33-4r}} \quad \Rightarrow \quad \text{general term}$$

The term contains $x^5 \Rightarrow 33 - 4r = 5 \Rightarrow 4r = 28 \Rightarrow \boxed{r = 7}$ then

Coefficient of term contains $x^5 = \left(-1\right)^7\, {}^{11}C_7 \times 2^{21-11} = -337920$

(ii) middle term T_6 , T_7

$$T_6 + T_7 = 0 \quad \Rightarrow \quad T_7 = -T_6 \quad , \quad \frac{T_7}{T_6} = -1 \quad \Rightarrow \quad \frac{11-6+1}{6} \times \frac{-4x^{-1}}{\frac{1}{2}x^3} = -1$$

$$\frac{-8}{x^4} = -1 \quad \Rightarrow \quad x^4 = 8 \quad \Rightarrow \quad x = \pm\sqrt[4]{8}$$

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3. (a) Let $Z = 2 - 2\sqrt{3}\, i \quad \Rightarrow \quad X = 2$ and $Y = -2\sqrt{3}$

$$r = |Z| = \sqrt{X^2 + Y^2} = \sqrt{4 + 12} = \boxed{4}$$

$$\tan\theta = \left|\frac{Y}{X}\right| = \frac{2\sqrt{3}}{2} = \sqrt{3} \quad \Rightarrow \quad \theta = 60^\circ \text{ and lies in } 4^{th} \text{ quad}$$

$$\text{P. amp.} = 360^\circ - 60^\circ = 300^\circ$$

$$Z = r\left[\cos\theta + i\ \sin\theta\right] = 4\left[\cos 300^\circ + i\ \sin 300^\circ\right]$$

(ii) $\left(a + i\ b\right)^2 = 4\left[\cos 300^\circ + i\ \sin 300^\circ\right]$

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$$a+i\,b = 2\left[\cos\frac{300^\circ + 360\times n}{2} + i\,\sin\frac{300^\circ + 360\times n}{2}\right]$$

Where n = 0, 1

<u>When n = 0</u>

$$a+i\,b = 2\left[\cos 150^\circ + i\,\sin 150^\circ\right]$$

$$a+i\,b = 2\left[-\frac{\sqrt{3}}{2} + \frac{1}{2}\,i\right] = -\sqrt{3} + i \quad\Rightarrow\quad \boxed{a = -\sqrt{3}\ ,\ b = 1}$$

<u>When n = 1</u>

$$a+i\,b = 2\left[\cos 330^\circ + i\,\sin 330^\circ\right]$$

$$a+i\,b = 2\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\,i\right] = \sqrt{3} - i \quad\Rightarrow\quad \boxed{a = \sqrt{3}\ ,\ b = -1} \quad \text{then}$$

$$\boxed{a = \pm\sqrt{3}\ ,\ b = \pm 1} \quad\Rightarrow\quad a+i\,b = \pm\left(\sqrt{3}+i\right)$$

$$X = \sqrt{3}\ ,\ Y = 1 \quad\Rightarrow\quad r = \sqrt{3+1} = 2$$

$$\tan\theta = \left|\frac{Y}{X}\right| = \frac{1}{\sqrt{3}} \quad\Rightarrow\quad \text{P. amp.} = 30^\circ$$

$$a+i\,b = \pm 2\left[\cos 30^\circ + i\,\sin 30^\circ\right]$$

$$\left(a+i\,b\right)^6 = 64\left[\cos 180^\circ + i\,\sin 180^\circ\right]$$

$$\left(a+i\,b\right)^6 = 64\left[-1 + 0\,i\right] = -64$$

(b) $(x-2)$ is a factor of Δ then $\Delta = 0$ when $x = 2$

$$\because \Delta = \begin{vmatrix} 1 & 5 & 2 \\ -3 & 7 & -6 \\ 5 & 2 & 2+k \end{vmatrix} = 0 \quad \text{By add } c_3 + \left(c_1 \times -2\right)$$

$$\therefore \Delta = \begin{vmatrix} \boxed{1} & 5 & 0 \\ -3 & 7 & 0 \\ 5 & 2 & k-8 \end{vmatrix} = 0$$

$$1\left[22(k-8) - (-23)(0)\right] = 0 \quad\Rightarrow\quad 22(k-8) = 0 \quad\Rightarrow\quad k-8 = 0 \quad\Rightarrow\quad \boxed{k = 8}$$

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4. (a) (i) It is parallel to the plane

(ii) any straight line intersecting it and is parallel to the second straight line

(iii) $= 4\sqrt{(15)^2 + \left(5\sqrt{3}\right)^2 + (10)^2} = 4 \times 20 = 80$ cm

---

(b) In $\Delta D'YD$

$\because DD' = AA' = 20$ cm

$\because m\left(\angle D'YD\right) = 90^\circ$ & $m\left(\angle D'DY\right) = 30^\circ$

$\therefore D'Y = \frac{1}{2}DD' = \frac{1}{2} \times 20 = 10$ cm

$\because D'X \perp \text{plane}(ABCD)$

$\therefore \overline{D'Y}$ Inc. whose proj. $\overline{YX}$

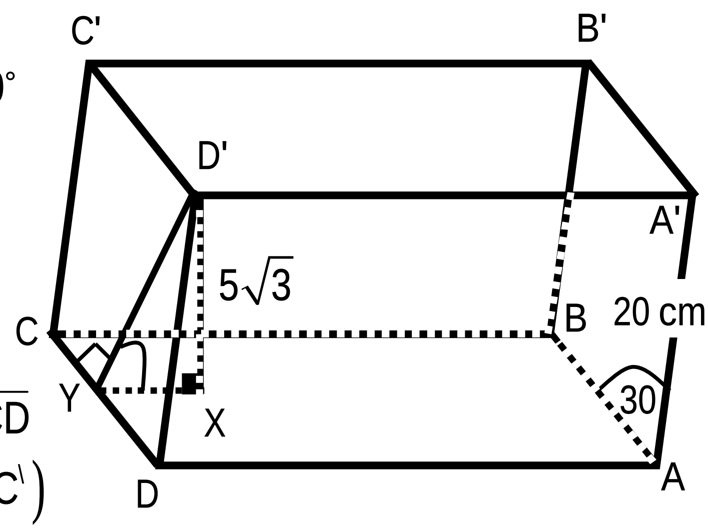
$\therefore$ Inc. $\overline{D'Y} \perp \overline{CD} \Rightarrow \therefore$ proj. $\overline{YX} \perp \overline{CD}$

$\because \overline{D'Y} \perp \overline{CD}$ and $\overline{D'Y} \subset \text{Plane}\left(CDD'C'\right)$

$\because \overline{XY} \perp \overline{CD}$ and $\overline{XY} \subset \text{Plane}(ABCD)$

$\therefore \angle D'YX$ is a plane angle of deh $\left(\angle D' - \overleftrightarrow{CD} - A\right)$

$\therefore \sin\left(D'YX\right) = \frac{D'X}{D'Y} = \frac{5\sqrt{3}}{10} \Rightarrow m\left(D'YX\right) = 60^\circ$



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5. (a) Given:-

$\overleftrightarrow{AB}$ Inclined on P , $\overleftrightarrow{AN} \perp P$, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

R. T. P:-

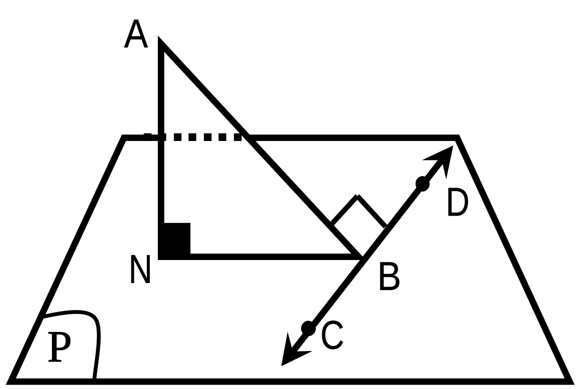
$\overline{BN} \perp \overleftrightarrow{CD}$

Proof:-

$\overline{AN} \perp P \Rightarrow \overline{AN} \perp \overleftrightarrow{CD}$ But $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

$\therefore \overleftrightarrow{CD} \perp$ to both of $\overline{AN}$ and $\overleftrightarrow{AB}$

$\therefore \overleftrightarrow{CD} \perp P(ABN) \Rightarrow \overleftrightarrow{CD} \perp \overline{NB}$



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(b) (i)∵ $\overleftrightarrow{AB}$ // plane(Mxy) and $\overleftrightarrow{AB}$ Ì P(ABD)

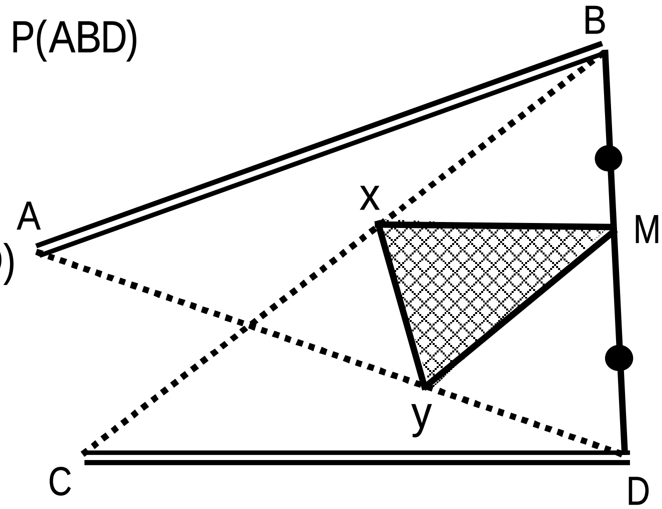
where P(Mxy) $\cap$ P(ABD) = $\overleftrightarrow{MY}$

\ $\overleftrightarrow{MY}$ // $\overline{AB}$

∵ $\overleftrightarrow{CD}$ // plane(Mxy) and $\overleftrightarrow{CD}$ Ì P(BCD)

∵ P(Mxy) $\cap$ P(BCD) = $\overleftrightarrow{Mx}$

\ $\overline{Mx}$ // $\overline{CD}$



(ii)<u>In ∆Mxy</u>

xy < yM + Mx ® (1) [Ineq. of triangle]

∵ $\overline{Mx}$ // $\overline{CD}$

\ $\frac{DM}{DB} = \frac{My}{AB}$ But $\frac{DM}{DB} = \frac{1}{2}$ Þ \ $\frac{My}{AB} = \frac{1}{2}$ Þ \ $\boxed{My = \frac{1}{2}AB}$ ® (2)

∵ $\overline{Mx}$ // $\overline{CD}$

\ $\frac{BM}{BD} = \frac{Mx}{CD}$ But $\frac{BM}{BD} = \frac{1}{2}$ Þ \ $\frac{Mx}{CD} = \frac{1}{2}$ Þ \ $\boxed{Mx = \frac{1}{2}CD}$ ® (3)

Sub. from (2) , (3) in (1)

$xy < \frac{1}{2}AB + \frac{1}{2}CD$ Þ $xy < \frac{1}{2}[AB + CD]$

---

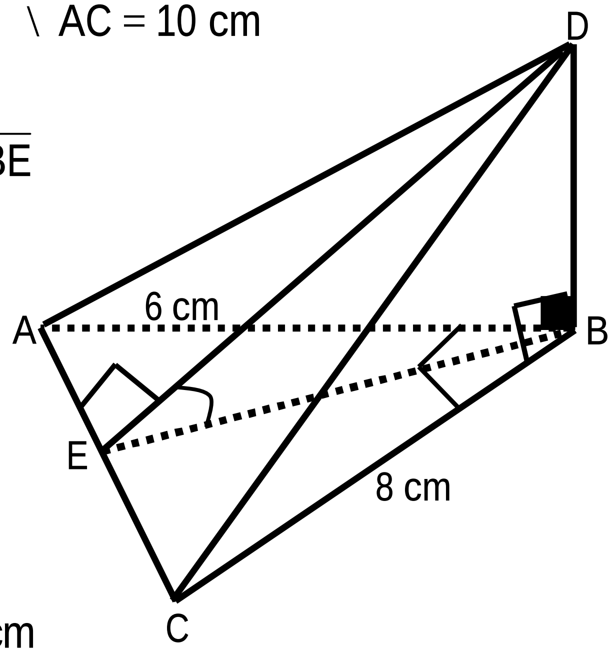
6. \ $(AC)^2 = (BC)^2 + (AC)^2$ Þ \ AC = 10 cm

∵ $\overline{DB}$ ^ Plane (ABC)

\ $\overline{DE}$ inclined to plane ABC its proj. $\overline{BE}$

∵ Inc. $\overline{DE}$ ^ $\overline{AC}$ Þ Proj. $\overline{BE}$ ^ $\overline{AC}$

\ $\overline{BE}$ ^ $\overline{AC}$ Þ \ $BE = \frac{8 ´ 6}{10} = 4.8$ cm



area of DDAC = 30

$\frac{1}{2}$ ´ AC ´ DE = 30 Þ DE = 6 cm

<u>In DDBE</u> ∵ m(DBE) = 90°

\ $(DB)^2 = (DE)^2 - (EB)^2$ Þ DB = 3.6 cm

\ $\tan(DEB) = \frac{3.6}{4.6} = \frac{3}{4}$

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